

Rotating Black Holes in Dilatonic Einstein-Gauss-Bonnet Theory

Burkhard Kleihaus, Jutta Kunz

Institut für Physik, Universität Oldenburg, D-26111 Oldenburg, Germany

Eugen Radu

School of Theoretical Physics – DIAS, 10 Burlington Road, Dublin 4, Ireland

Department of Computer Science, National University of Ireland Maynooth, Maynooth, Ireland

(Dated: January 17, 2011)

We construct generalizations of the Kerr black holes by including higher curvature corrections in the form of the Gauss-Bonnet density coupled to the dilaton. We show that the domain of existence of these Einstein-Gauss-Bonnet-dilaton (EGBd) black holes is bounded by the Kerr black holes, the critical EGBd black holes, and the singular extremal EGBd solutions. The angular momentum of the EGBd black holes can exceed the Kerr bound. The EGBd black holes satisfy a generalised Smarr relation. We also compare their innermost stable circular orbits with those of the Kerr black holes and show the existence of differences which might be observable in astrophysical systems.

PACS numbers: 04.70.-s, 04.70.Bw, 04.50.-h

Introduction.— Black holes (BHs) represent some of the most fascinating objects in nature. In the past decades, observational evidence for their existence has increased tremendously, making BHs essential ingredients of modern astrophysics on all scales, from stellar binaries to galaxies and active galactic nuclei.

The existence of BHs is an important prediction of general relativity, with the astrophysically most relevant Kerr solution found almost half a century ago. String theory, however, suggests the existence of higher curvature corrections to the Einstein equations, which may lead to new qualitative features of the solutions. One of the simplest four-dimensional models with higher-curvature terms is the so-called Einstein-Gauss-Bonnet-dilaton (EGBd) gravity, which is obtained by adding to the Einstein action the four-dimensional Euler density multiplied by the dilaton exponent together with the dilaton kinetic term. EGBd gravity has the attractive features that the equations of motions are still of second order and that the theory is ghost-free. Although the BHs in this theory cannot be found in analytical form, the static solutions were extensively studied perturbatively [1], [2], and numerically [3], [4], [5]. These BHs contain classical non-trivial dilaton fields and thus can evade the classical “no-scalar-hair” theorem.

However, astrophysical black holes are expected to be highly spinning, and nearly extreme rotating black holes have been observed in the sky [6]. Rotating solutions in EGBd theory have not yet been constructed, except in lowest order perturbation theory [7]. Therefore, we here address the question on how the GBd term affects the properties of rotating BHs. We construct the EGBd generalizations of the Kerr solution within a nonperturbative approach by directly solving the field equations with suitable boundary conditions. We exhibit their domain of existence and study their geodesics.

The model.— We consider the following low-energy effective action for the heterotic string

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - \frac{1}{2} (\partial_\mu \phi)^2 + \alpha e^{-\gamma \phi} R_{\text{GB}}^2 \right], \quad (1)$$

where ϕ is the dilaton field with coupling constant γ , α is a numerical coefficient given in terms of the Regge slope parameter, and $R_{\text{GB}}^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2$ is the GB correction.

We employ the usual Lewis-Papapetrou ansatz [8] for a stationary, axially symmetric spacetime with two Killing vector fields $\xi = \partial_t$, $\eta = \partial_\varphi$. In terms of the spherical coordinates r and θ , the isotropic metric reads [9]

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -f dt^2 + \frac{m}{f} (dr^2 + r^2 d\theta^2) + \frac{l}{f} r^2 \sin^2 \theta \left(d\varphi - \frac{\omega}{r} dt \right)^2, \quad (2)$$

where f , m , l and ω are functions of r and θ , only. The event horizon of these stationary black holes resides at a surface of constant radial coordinate $r = r_{\text{H}}$, and is characterized by the condition $f(r_{\text{H}}) = 0$. At the horizon we impose the boundary conditions $f = m = l = 0$, $\omega = \Omega_{\text{H}} r_{\text{H}}$, and $\partial_r \phi = 0$, where Ω_{H} is the horizon angular velocity. The boundary conditions at infinity, $f = m = l = 1$, $\omega = 0$, $\phi = 0$, ensure that the solutions are

asymptotically flat with a vanishing dilaton. Axial symmetry and regularity impose the boundary conditions on the symmetry axis ($\theta = 0$), $\partial_\theta f = \partial_\theta l = \partial_\theta m = \partial_\theta \omega = 0$, $\partial_\theta \phi = 0$, and, for solutions with parity reflection symmetry, agree with the boundary conditions on the $\theta = \pi/2$ -axis. The absence of conical singularities implies also $m = l$ at $\theta = 0$.

The mass M , the angular momentum $J = aM$, and the dilaton charge D are obtained from the asymptotic expansion via

$$f \rightarrow 1 - \frac{2M}{r}, \quad \omega \rightarrow \frac{2J}{r^2}, \quad \phi \rightarrow -\frac{D}{r}. \quad (3)$$

Of interest are also the properties of the horizon. The surface gravity κ_{sg} is obtained from [8] $\kappa_{\text{sg}}^2 = -1/4(D_\mu \chi_\nu)(D^\mu \chi^\nu)$ where the Killing vector $\chi = \xi + \Omega_H \eta$ is orthogonal to and null on the horizon. Expansion near the horizon in $\delta = (r - r_H)/r_H$ yields to lowest order $f = \delta^2 f_2(\theta)$, $m = \delta^2 m_2(\theta)$, showing that the surface gravity is indeed constant on the horizon. The Hawking temperature T_H of the black hole is

$$T_H = \frac{\kappa_{\text{sg}}}{2\pi} = \frac{1}{2\pi r_H} \frac{f_2(\theta)}{\sqrt{m_2(\theta)}}. \quad (4)$$

The entropy of these BHs can be written in Wald's form [10] as an integral over the event horizon

$$S = \frac{1}{4} \int_{\Sigma_h} d^3x \sqrt{h} (1 + 2\alpha e^{-\gamma\phi} \tilde{R}), \quad (5)$$

where h is the determinant of the induced metric on the horizon and \tilde{R} is the event horizon curvature.

The event horizon properties and the global charges are related through the Smarr mass formula

$$M = 2T_H S + 2\Omega_H J - \frac{D}{2\gamma}, \quad (6)$$

analogous in form to the mass formula of hairy black holes [11]. This Smarr relation is obtained by starting from the Komar expressions, and making use of the equations of motion, and the expansions at the horizon and at infinity.

The solutions.— We solve the set of five second order coupled non-linear partial differential equations for the functions f, l, m, ω and ϕ numerically, subject to the above boundary conditions, employing a compactified coordinate $x = (r - r_H)/(1 + r)$. As initial guess we use the static BH solutions in EGBd theory with a given $\alpha > 0$. By increasing Ω_H from zero, we obtain rotating BH solutions, whose mass M , dilaton charge D and angular momentum J are determined from their asymptotic behaviour (see eq. (3)). For all numerical calculations we choose the dilaton coupling $\gamma = 1$.

Let us first address the domain of existence of these EGBd BHs. In Fig. 1 (left) we show the scaled horizon area $a_H = A_H/M^2$ as function of the scaled angular momentum $j = J/M^2$. The domain of existence is indicated by the shaded area. The Kerr BHs are all mapped to a single curve, with the set of Schwarzschild BHs mapped to the point $a_H = 1$, $j = 0$. For these Einstein BHs the scaled entropy $s = S/M^2$ and area are proportional, $s = a_H/4$. We observe that the Einstein BHs form one of the boundaries of the domain of existence. With respect to the scaled entropy s , the Kerr values form the lower boundary in the range of existence of Kerr solutions, i.e., for $j \leq 1$.

The left border of the domain of existence ($j = 0$) is formed by the static EGBd BHs. These are known to exist in the range $4 \leq s \leq 4.8$, or in terms of the scaled area a_H , exhibited in Fig. 1 (right), in the range $0.85 \leq a_H \leq 1$ [3]. In this range the scaled area a_H of the static EGBd BHs increases monotonically with increasing mass to the Schwarzschild value $a_H = 1$, while the scaled entropy decreases monotonically with increasing mass. The critical static solution has the smallest scaled area and the largest scaled entropy possible for a static EGBd BH. The occurrence of this bound is seen in the horizon expansion of the dilaton field, where it originates when a square root vanishes as the critical horizon size is reached [3].

Analogous to the static case, such critical solutions exist also for the rotating EGBd BHs. These critical black holes form the upper boundary of the domain of existence, when considered in terms of the scaled entropy versus the scaled angular momentum, as seen in Fig. 1 (left). Also shown in the figure are families of EGBd BHs with fixed scaled horizon angular velocity $\Omega_H \alpha^{1/2}$, which all begin at their respective critical BH solution,

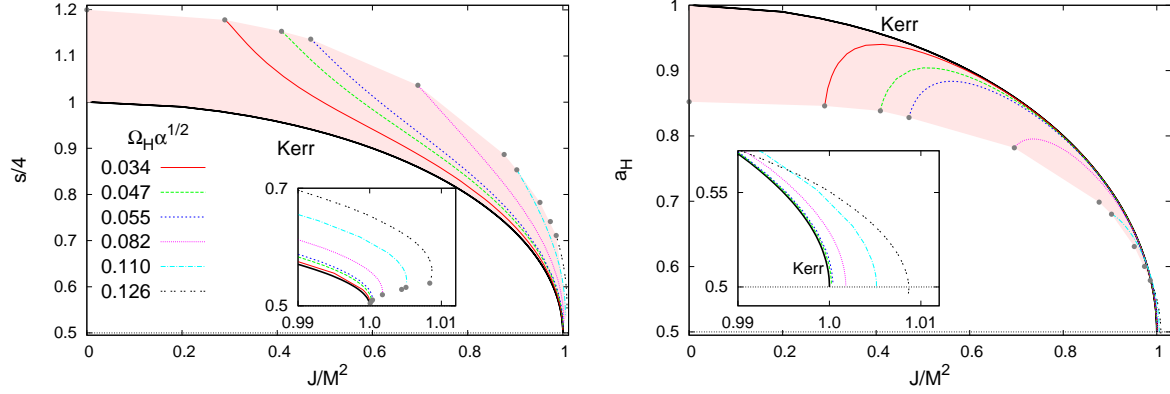


FIG. 1: The scaled entropy $S = S/M^2$ (left) and scaled horizon area $a_H = A_H/M^2$ (right) versus the scaled angular momentum $j = J/M^2$ for families of EBGd BHs with fixed scaled horizon angular velocity $\Omega_H \alpha^{1/2}$. The shaded area indicates the domain of existence. The dots in the inset of the left panel correspond to extrapolated values.

corresponding to the smallest scaled angular momentum for this value of $\Omega_H \alpha^{1/2}$. But where do these curves of fixed $\Omega_H \alpha^{1/2}$ end?

The family of Kerr BHs ends at the extremal Kerr solution, which precisely saturates the Kerr bound for the scaled angular momentum, $j_{\text{Kerr}} \leq 1$. Interestingly, the EBGd BHs can exceed the Kerr bound. Indeed, we find $j \leq 1.02$ as highlighted in the inset of Fig. 1 (right). We observe that the families of EBGd BHs with fixed scaled horizon angular velocity $\Omega_H \alpha^{1/2}$ all end, when a respective singular extremal EBGd solution is reached. These singular extremal EBGd solutions form the final part of the boundary of the domain of existence of the EBGd BHs, as seen in the inset of Fig. 1 (right).

Extremal Kerr BHs have vanishing temperature and vanishing isotropic horizon radius. Likewise, the families of EBGd BHs end in solutions with vanishing temperature and vanishing isotropic horizon radius. The vanishing of the temperature at the end point of these curves is seen in Fig. 2 (left), where the scaled temperature $T_H M$ is exhibited versus the scaled angular momentum. Clearly, these endpoints form the lower part of the boundary of the domain of existence for $j > 1$, while the other boundaries are formed by the critical EBGd BHs, the Kerr BHs.

However, unlike the extremal Kerr solution, the extremal EBGd solutions are not regular. While the (isotropic) metric functions tend to well defined limiting functions, with the isotropic horizon radius and the surface gravity approaching zero, the dilaton field diverges in this limit at the poles of the BH horizon, making the extremal solutions singular.

The nonexistence of regular extremal solutions is also seen when, following *e.g.* [12], one attempts to construct the corresponding near-horizon geometries with an isometry group $SO(2,1) \times U(1)$. Again, the dilaton field at the horizon is found to diverge at the poles. Indeed, this holds true independent of the value of the dilaton coupling constant, also a perturbative solution exists for small α .

At the outermost point of the domain of existence, where the scaled angular momentum j reaches its maximal value, the branches of critical EBGd BHs and singular extremal solutions merge and end. Here the scaled horizon angular velocity reaches its maximal value, $\Omega_H \alpha^{1/2} \approx 0.135 - 0.14$.

Summarizing the black hole properties, we note that for a EBGd theory, specified by the coupling constant α and $\gamma = 1$, the regular rotating EBGd BHs are found within their domain of existence delimited by the Kerr BHs, the critical EBGd BHs and the singular extremal EBGd solutions. For a given mass and angular momentum, an EBGd BH has a higher entropy and temperature than a Kerr black hole. Moreover, EBGd BHs exist beyond the Kerr bound. The high quality of the numerical solutions is seen from the Smarr relation (6), which is satisfied for the EBGd BH families within an accuracy of 10^{-5} .

Geodesics.— Let us end by considering possible observational effects for such EBGd BHs, as addressed before only for static and slowly rotating solutions [7]. For instance, by studying the geodesic motion around such BHs the location and orbital frequency of the innermost-stable-circular-orbit (ISCO) was found to differ up

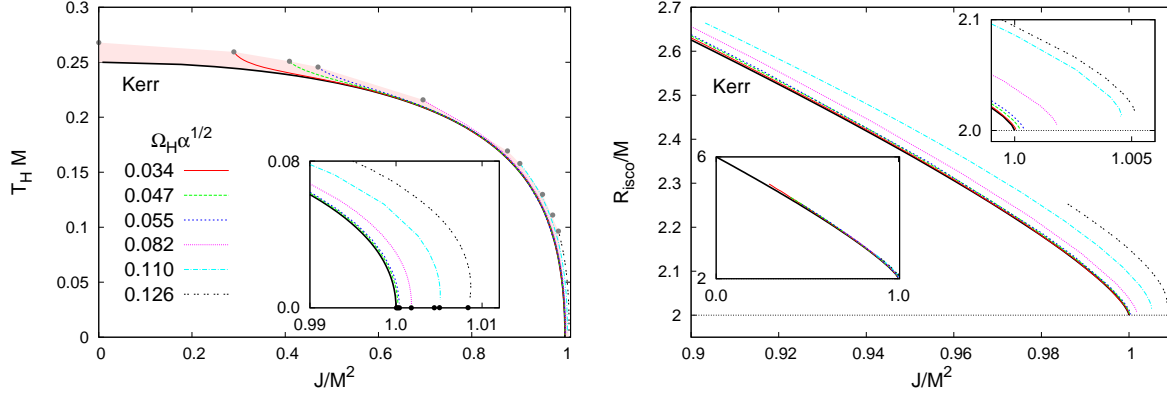


FIG. 2: The scaled temperature $T_H M$ (left) and scaled circumferential ISCO radius R_{ISCO}/M for dilaton matter coupling constant $\beta = 0.5$ (right) versus the scaled angular momentum $j = J/M^2$ for families of EBGd BHs with fixed scaled horizon angular velocity $\Omega_H \alpha^{1/2}$. The shaded area indicates the domain of existence.

to a few percent from the Kerr values, depending on the values of the global charges of the respective solutions [7].

Here we extend this study to the astrophysically relevant regime of fast rotating BHs. Restricting to motion in the equatorial plane, we consider timelike geodesics, with a Lagrangian $2\mathcal{L} = e^{-2\beta\phi} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = -1$, where the constant β fixes the coupling between the matter and the dilaton field (with $\beta = 0.5$ in heterotic string theory; also, a superposed dot denotes the derivative with respect to the affine parameter along the geodesics). Taking into account the existence of two conserved quantities associated with the Killing vectors ξ and η , one finds the equation $\dot{r}^2 = V(r)$, with the potential $V(r)$ depending on the dilaton and the metric functions at $\theta = \pi/2$. For circular orbits $V(r) = V'(r) = 0$; stability of the orbits then requires that the second derivative of the effective potential is negative.

We exhibit in Fig. 2 (right) the scaled circumferential ISCO radius R_{ISCO}/M versus the scaled angular momentum $j = J/M^2$ for families of EBGd BHs with fixed scaled horizon angular velocity $\Omega_H \alpha^{1/2}$, choosing the dilaton matter coupling $\beta = 0.5$. The maximal values of R_{ISCO}/M correspond to critical EBGd BHs, while the Kerr values and the singular extremal EBGd values form the lower boundary for the scaled ISCO radius R_{ISCO}/M (for this β).

For extremal Kerr solutions the ISCO radius is known to tend to the horizon radius. Since for extremal Kerr solutions the circumferential horizon radius corresponds to twice the Boyer-Lindquist radius, the scaled ISCO radius R_{ISCO}/M tends to the value two in the extremal Kerr limit. For the EBGd solutions we likewise observe, that the ISCO radius tends to the horizon radius in the singular extremal limit. Here also the dependence on the dilaton coupling β vanishes. In contrast, generically the scaled ISCO radius decreases with decreasing β towards and (for small β and j) below the corresponding Kerr value. For large scaled angular momentum ($j \leq 1$) the deviation of R_{ISCO}/M from the Kerr value can be as much as 10% (for $\beta = 0.5$). Similarly, the orbital frequencies exhibit the largest deviations from the Kerr frequencies in this range and can amount to 60%. Effects of this size for fast rotating black holes might be observable in astrophysical systems.

On the more theoretical side we note, that by including gauge fields further interesting rotating hairy black holes should be generated, representing new solutions of the low energy effective action of string theory.

[1] S. Mignemi and N. R. Stewart, Phys. Rev. D **47** (1993) 5259 [arXiv:hep-th/9212146].

[2] S. Mignemi, Phys. Rev. D **51** (1995) 934 [arXiv:hep-th/9303102].

[3] P. Kanti, N. E. Mavromatos, J. Rizos, K. Tamvakis and E. Winstanley, Phys. Rev. D **54** (1996) 5049 [arXiv:hep-th/9511071].

- [4] T. Torii, H. Yajima and K. i. Maeda, Phys. Rev. D **55** (1997) 739 [arXiv:gr-qc/9606034].
- [5] S. O. Alexeev and M. V. Pomazanov, Phys. Rev. D **55** (1997) 2110 [arXiv:hep-th/9605106].
- [6] J. E. McClintock, R. Shafee, R. Narayan, R. A. Remillard, S. W. Davis and L. X. Li, Astrophys. J. **652** (2006) 518 [arXiv:astro-ph/0606076].
- [7] P. Pani and V. Cardoso, Phys. Rev. D **79** (2009) 084031 [arXiv:0902.1569 [gr-qc]].
- [8] R. M. Wald, General Relativity (University of Chicago Press, Chicago, 1984)
- [9] B. Kleihaus and J. Kunz, Phys. Rev. Lett. **86**, 3704 (2001) [arXiv:gr-qc/0012081].
- [10] R. M. Wald, Phys. Rev. D **48** (1993) 3427 [arXiv:gr-qc/9307038].
- [11] B. Kleihaus, J. Kunz and F. Navarro-Lerida, Phys. Rev. Lett. **90**, 171101 (2003) [arXiv:hep-th/0210197].
- [12] D. Astefanesei, K. Goldstein, R. P. Jena, A. Sen and S. P. Trivedi, JHEP **0610** (2006) 058 [arXiv:hep-th/0606244].